Wavelet Transform Embeddings in Mesh Architectures

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Abstract

To efficiently use wavelet transforms in parallel mesh architectures, we need to identify efficient embeddings of wavelet transform coefficients into such meshes. We consider two forms of 2D wavelet transform embedding into 2D meshes (with and without reconfigurability) and compare time performances for these embeddings over classes of image processing algorithms — demonstrating the superiority of one of these embeddings.

1 Introduction

Wavelet transforms have excited a lot of interest as a new form of multiresolution representation for 1D signals and 2D images [4]. Although pyramid architectures are a natural fit to multiresolution approaches, many mesh architectures exist [1] and are likely to be more readily available in the future. Further, reconfigurable 2D meshes have been shown to emulate pyramid architectures well [3], Chaps. 2, 6. Thus, mesh architectures appear to be desirable targets for 2D wavelet based algorithm development. A principal question is how to efficiently embed wavelet transform coefficients into given target meshes. This impacts on the time complexity of various parallel algorithms, typically by affecting the communication overhead. In this note we consider certain such embeddings and the communication overhead for several algorithm classes.

2 2D Mesh Embeddings

Two 2D mesh models are considered: (1) MESH2 is a traditional mesh connected computer model (MCC) including 4-neighbor links and (2) RMESH2 is a reconfigurable 2D mesh with separate row and column busses similar in capability to the polymorphic torus ([3], Chap. 2). Typical reconfigurations used divide a given row or column bus into distinct busses. Two 2D mesh embeddings are considered as illustrated in Figure 1. The original image is at level 0 with size $n \times n$ ($n=2^s$) and successive levels, $i > 0$, refer to the lower resolution image representations characteristic of the wavelet transform [4]. This representation is generated recursively by applying special filters (typically high pass) to the original image generating a new set of images at half the resolution, i.e. level 1 *detail* images D_1^1, D_1^2, D_1^3 in Figure 1a. The low pass image (A) at level 1 has been replaced by recursive application of this process which in Figure 1a stops at level 3. In the embedding illustrated in Figure 1a (following [4]) the detail images D_i^1, D_i^2 and D_i^3 at level i and the lower resolution version of the image, A, are concentrated into subblocks in the 2D mesh — denoted *Block-Concentrated* (BC2). This embedding puts detail images and A in contiguous blocks but the four components corresponding to a given local region in the image are separated substantially so that local operators using all components would sustain a large communication overhead accessing these components. Thus, a second embedding has been developed, denoted *Distributed* (DIST2) and illustrated in Figure 1b, where the individual elements of A, a, and the individual elements of the detail images at level i, d_i^1, d_i^2 and d_i^3, are distributed uniformly over the mesh. The DIST2 embedding is defined recursively, i.e. at level i 2×2 regions of the original image (or A for $i > 0$) are replaced with,

$$\begin{matrix} a & d_i^1 \\ d_i^2 & d_i^3 \end{matrix} .$$

This continues recursively to the desired level, e.g. level 3 in Figure 1b. The communication distance between the PE's to which specific detail image elements at level i are mapped is 2^i for DIST2 and 2^{s-i+1} for BC2. Thus for algorithms requiring access to all four image components, BC2 has higher communication cost at lower levels and lower communication cost at higher levels while the opposite holds for DIST2.

3 Time Performance Results

In time performance evaluations one communication step is assessed for: (1) Each filter coefficient broadcast; (2) Each read of a datum from 4-neighbor PE's or from the row or column busses (RMESH2 only); and (3) Each reconfiguration of RMESH2. Figure 2 reports *communication overhead* (total commu-

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nication steps divided by n) for wavelet decomposition and reconstruction as defined in [4]. We assume separable filters (8×8) realized with a convolution algorithm similar to [5]. For Figure 2a the transforms are computed till the lowest resolution image is the size of the filter, i.e. 8×8. For MESH2 BC2 and DIST2 have similar overhead for decompostion but DIST2 has substantially higher overhead for reconstruction. For RMESH2 DIST2 is able to fully utilize reconfigurability achieving time performance essentially independent of image space size; whereas, BC2 performance cannot be improved by reconfigurability. Often in multiresolution approaches we will realize only a few levels and in Figure 2b the transforms are performed only to 3 levels of resolution for each image space size. The principal change is that DIST2 performance is typically superior to that for BC2 for *either* mesh model. Thus, where one can avoid multiresolution representations with many levels the DIST2 embedding appears to be superior for regular mesh architectures; whereas, DIST2 is uniformly superior for reconfigurable meshes. Similar results have been achieved for general classes of local operators and models of top-down "planning" processes [2]. Further, the superiority of the DIST2 embedding has been demonstrated for the embedding of larger images into smaller meshes; and the DIST2 embedding notions have been extended to 3D images and meshes [2].

4 Conclusions

Embeddings of wavelet transform coefficients into 2D mesh architectures have been considered and evaluated for classes of image processing algorithms. A particular form of distributed embedding has been found to be superior for most test instances in an ordinary 2D mesh and to be ideal for reconfigurable 2D meshes.

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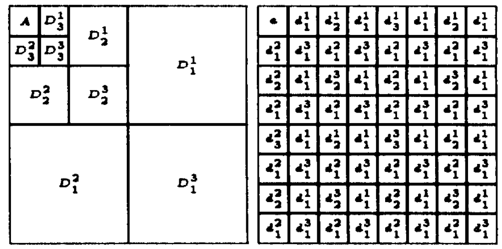
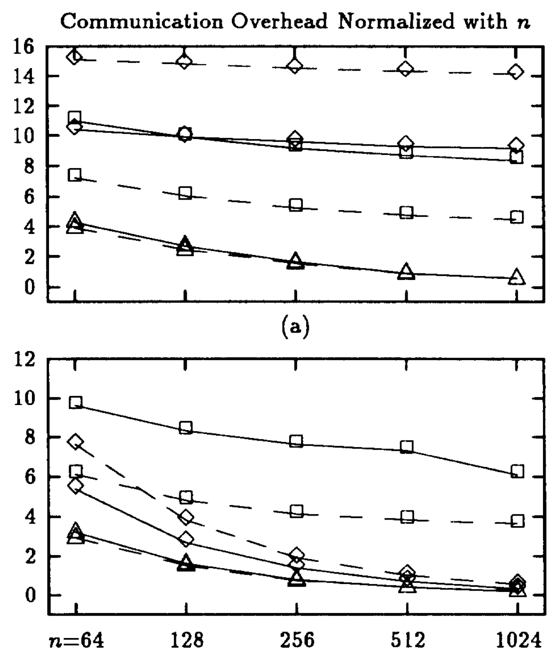


(a) (b)

Figure 1. Wavelet transform embeddings in a 2D mesh: (a) BC2; (b) DIST2.



(b)

Figure 2. Communication overhead for wavelet decomposition (solid lines), and reconstruction (dashed lines); □–BC2-MESH2/BC2-RMESH2, ◇–DIST2-MESH2, △–DIST2-RMESH2. (a) Taken to 8×8 lowest resolution, (b) taken for only 3 levels. (Filter size is 8×8)